

HEAT AND MASS TRANSFER ANALYSIS OF STEADY MHD FREE CONVECTIVE FLUID FLOW PAST AN INCLINED STRETCHING POROUS SHEET WITH VISCOUS DISSIPATION AND RADIATION

K. V. CHANDRA SEKHAR¹ & V. MANJULA²

¹Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India

²Research Scholar, Konerulakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India

ABSTRACT

The present paper describes MHD free convection flow past inclined porous stretching sheet under the influence of viscous dissipation and Radiation effects. Similarity transformations are used to reduce non linear partial differential equations to ordinary differential equations. Approximate solutions have been derived for velocity, temperature and concentration, using R-k method of fourth order along with shooting technique. Graphical analysis has been done to identify influences of different physical parameters on velocity, temperature and concentration.

KEYWORDS: Angle of Inclination, Magnetic Field, Eckert Number, Radiation Parameter, Porosity & Chemical Reaction Parameter

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1. NOMENCLATURE

α	Angle of inclination
B_0	Magnetic field strength
σ	Electrical conductivity
σ_1	Stefan-Boltzmann constant
ρ	Density of the fluid
ψ	Stream function
u	Velocity component along x direction
v	Velocity component along y direction
T	Fluid temperature
θ	Dimensionless fluid temperature
η	Dimensionless distance normal to the sheet
T_∞	Free stream temperature

T_w	Uniform sheet temperature
κ	Thermal conductivity
c_p	Specific heat at constant pressure
Q_0	Volumetric rate of heat generation/absorption
q_r	Heat flux
C	Concentration
C_∞	Free stream concentration
ϕ	Dimensionless concentration
C_w	Concentration of the sheet at wall
κ	Thermal conductivity
κ_1	Mean absorption coefficient
ν	Kinematic viscosity
g_0	Acceleration due to gravity
k	Porosity

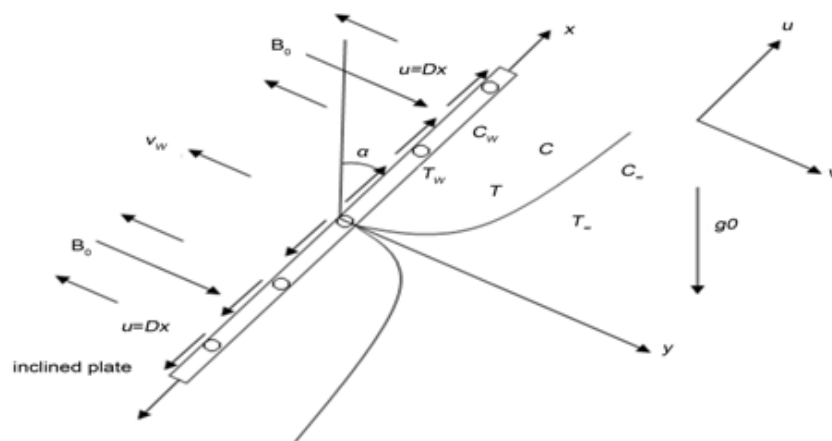
2. INTRODUCTION

Investigation of radiation interaction with natural convection got good significance in recent years. This is due to the presence of thermal radiation in the surface heat transfer. In technical processes like paper production, crystal growing, glass blowing, aerodynamics, we can observe boundary layer flow on continuous surfaces. Because of this, heat transfer analysis plays vital role in industries. Ostrach [9] studied the laminar convection flow and heat transfer of fluid with constant wall temperature. Hossain and Takhar [5] analyzed the effect of radiation effects on free convection flow of a gas past a semi infinite flat plate. Ali et al. [1] examined the radiation effect on natural convection flow over a vertical surface in a gray gas. Later, Mansour [8] investigated the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal, continuous moving sheet with suction/injection. Sattar and Kalim [10] analyzed the combined unsteady free convection dynamic boundary layer and thermal radiation boundary layer on a semi-infinite vertical plate by using the Roseland diffusion approximation. Chen [4] investigated natural convection flow over a permeable surface with variable wall temperature and concentration. The viscous dissipation plays an important role in the cooling of nuclear reactors. K. V. Chandrasekhar [3] analyzed the heat transfer of MHD flow over exponentially stretching sheet. Viscous dissipation effects on non-linear MHD flow in a porous medium over a stretching porous surface have been analyzed by S. P. Anjali Devi and B. Ganga [2]. Jha and Ajibade [6] studied the effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions. Hassan et al [7] considered the

combined effect of viscous dissipation and radiation on free convection flow an inclined stretching sheet. In the present analysis chemical reaction was included, in addition to the problem of Hassan et al [7].

3. MATHEMATICAL MODEL

A steady-state two-dimensional heat and mass transfer flow of an electrically conducting viscous incompressible chemically reacting fluid is considered along an isothermal permeable stretching sheet making an angle of inclination α to the vertical embedded in a porous medium with heat generation/absorption. A strong magnetic field is applied along the y-axis direction. Here the effect of the induced magnetic field is assumed to be small enough in comparison to the applied magnetic field. The electrical current flowing in the fluid will develop induced magnetic field if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting. Hence, only the applied magnetic field of strength B_0 plays a role which gives rise to magnetic forces in x-direction, $F = \frac{\sigma B_0^2}{u}$ in x – direction, where σ is the electrical conductivity assumed to be directly proportional to the x- translational velocity. Two equal and opposite forces are introduced along the x- axis so that the sheet is stretched keeping the origin fixed as shown in Figure.



Geometry of the Problem

The fluid is assumed to be gray, absorbing emitting radiation, but non scattering medium and the Roseland approximation is used to describe the radiation heat flux in the energy equation. The radiative heat flux in the x-direction is negligible in the y-direction. The plate temperature and concentration are initially raised to T_w and C_w respectively, which are thereafter maintained constant. The ambient temperature of the flow is T_∞ and the concentration of the uniform flow is C_∞ . The flow and heat transfer in the presence of radiation are described as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g_0 \beta (T - T_\infty) \cos \alpha - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu u}{k} - \frac{bu^2}{k} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - kr^2 (C - C_\infty) \quad (4)$$

The corresponding boundary conditions are

$$u = Dx, v = v_w(x), T = T_w, C = C_w \text{ at } y=0,$$

$$u = 0, T = T_\infty, C = C_\infty \text{ As } y \rightarrow \infty. \quad (5)$$

By using Rosseland approximation, takes the form

$$q_r = -\frac{4\sigma}{3\kappa_1} \frac{\partial T^4}{\partial y} \quad (6)$$

It is assumed that the temperature difference within the flow is sufficiently so that may be expressed as a linear function of temperature.

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^3 \quad (7)$$

Using (7) in (3) gives

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16\sigma_1 T_\infty^3}{3\rho c_p \kappa_1} \frac{\partial^2 T}{\partial y^2} \quad (8)$$

4. SIMILARITY ANALYSIS

To obtain the similarity solution, the following similarity transformations are used.

$$\eta = y \sqrt{\frac{D}{\nu \text{Re}_x}}, \psi = x \sqrt{\frac{\nu \text{Re}_x}{D}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (9)$$

Using the above transformations equations (2), (8) and (4) takes the form

$$f''' + \frac{\text{Re}_x}{D} f(\eta) f''(\eta) + \left(M + \frac{k}{Da} \right) f'(\eta) + \left(1 + \frac{F_s}{Da} \right) \frac{\text{Re}_x}{D} f'(\eta)^2 - \gamma \text{Re}_x D \theta \cos \alpha \quad (10)$$

$$\theta''(\eta) + Pn \text{Re}_x Q \theta + \frac{Pn Ec}{D^2} f''(\eta)^2 + \frac{Pn \text{Re}_x}{D} \theta'(\eta) f(\eta) = 0 \quad (11)$$

$$\phi''(\eta) + \frac{Sc \text{Re}_x}{D} f(\eta) \phi(\eta) - kr^2 \phi = 0 \quad (12)$$

Where,

$$\gamma = \frac{Gr_x}{Re_x^2} = \frac{g_0 \beta (T - T_\infty)}{D^2 x}, M = \frac{\sigma B_0^2}{\rho D}, Da = \frac{k}{x^2}, Re_x = \frac{x^2 D}{\nu}, F_s = \frac{b}{x},$$

$$Pr = \frac{\mu c_p}{\kappa}, Q = \frac{Q_0}{\rho c_p D}, Ec = \frac{D^2 x^2}{c_p (T - T_\infty)}, N = \frac{\kappa \kappa_1}{4 \sigma_1 T_\infty^3}, Pn = \frac{3 N Pr}{4 + 3 N}, Sc = \frac{\nu}{D_m} \quad (13)$$

The boundary conditions (5) transformed as

$$f(\eta) = Fw, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1 \text{ at } \eta = 0 \quad (14)$$

$$f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \text{ As } \eta \rightarrow \infty \text{ Where } Fw = \frac{-V_w}{\sqrt{D\nu}}$$

Numerical Computation

Numerical solution for the nonlinear ode (10), (11) and (12) along with boundary conditions (14) are derived using R - K method of fourth order through MATLAB CODE.

5. RESULTS AND DISCUSSIONS

For the discussion of results of flow field, the numerical computations were presented in the form of profiles of non dimensional velocity temperature and concentration Numerical computations have been carried out for different values of the suction parameter (Fw), Magnetic field parameter (M), permeable parameter (k) angle of inclination (α), heat source parameter (Q), radiation parameter (N), Reynolds number (Re), Eckert number (Ec), chemical reaction parameter (κc), and Schmidt number (Sc). The results are in good agreement with That of Hasan et al [7]. The results of the numerical solution for the velocity, temperature and concentration profiles are depicted in Figures 1 – 14.

Velocity profiles for different values of α , k , Re , M and Fw were shown in Figure –1to 5. From Figure – 1, It was observed that increase of angle of inclination results in decrease in velocity, but as we move away from the sheet reverse trend was observed. This is due to decrease in buoyancy force. Figure – 2 depicts that increase of permeability decreases the velocity. But far away from the sheet the effect is not significant. Figure – 3 depicts velocity decreases with increase of Reynolds number. Figure – 4 shows the influence of Magnetic parameter on velocity. It can be seen that flow is slow near the sheet with increase of M . But later a reverse trend was observed. Figure – 5 describes effect of suction parameter on the flow. It is noticed that velocity decreases with increase of suction.

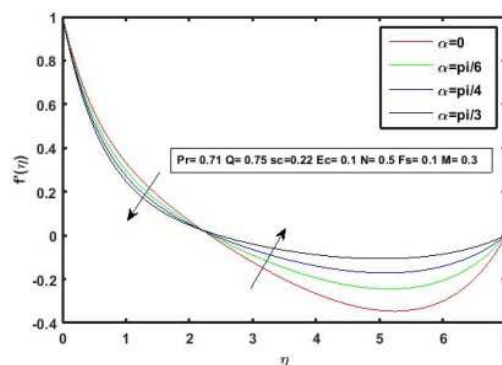


Figure 1: Velocity Profiles for Different Values of Angle of Inclination

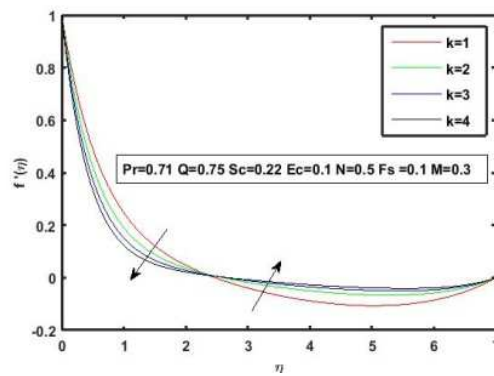


Figure 2: Velocity Profiles for Different Values of Porosity

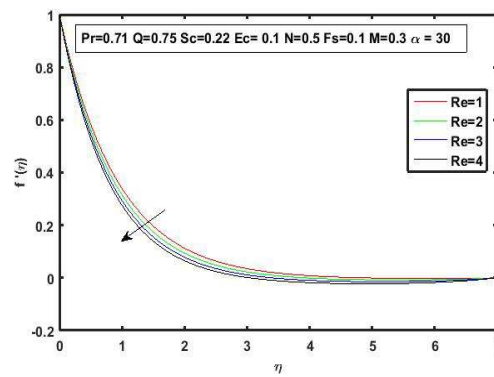


Figure 3: Velocity Profiles for Different Values of Reynolds Number

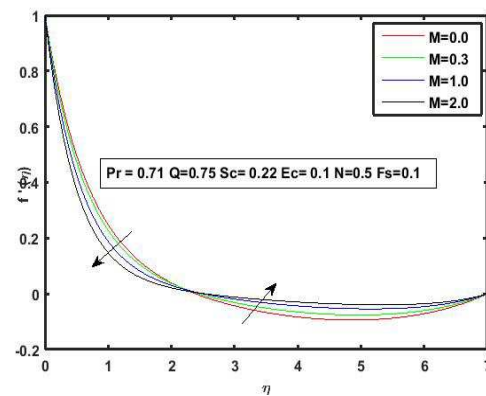


Figure 4: Velocity Profiles for Different Values of Magnetic Parameter

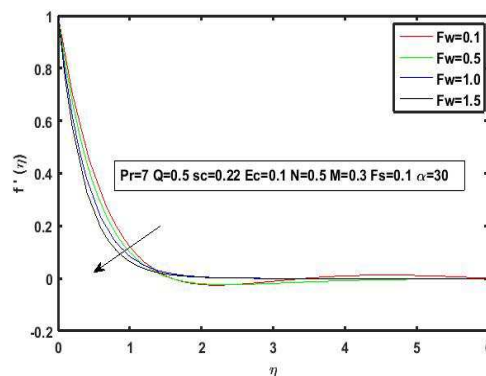


Figure 5: Velocity Profiles for Different Values of Suction Parameter

Figures 6 - 10 shows the impact of Radiation, Heat source, Reynolds number, suction parameter and Ekert number. It was observed that increase of Heat source, Reynolds number, Ekert number leads to increase in temperature, but decreases with increase of Radiation parameter, and suction.

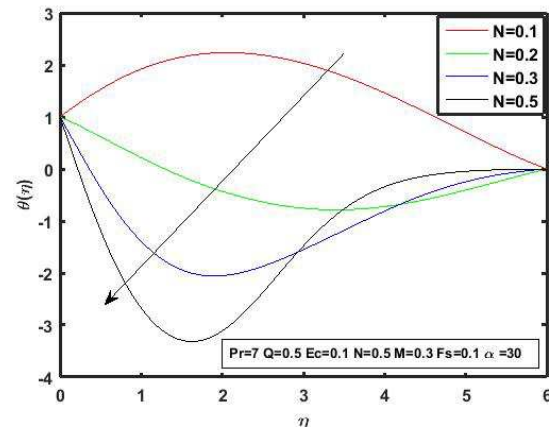


Figure 6: Temperature Profiles for Different Values of Radiation Parameter

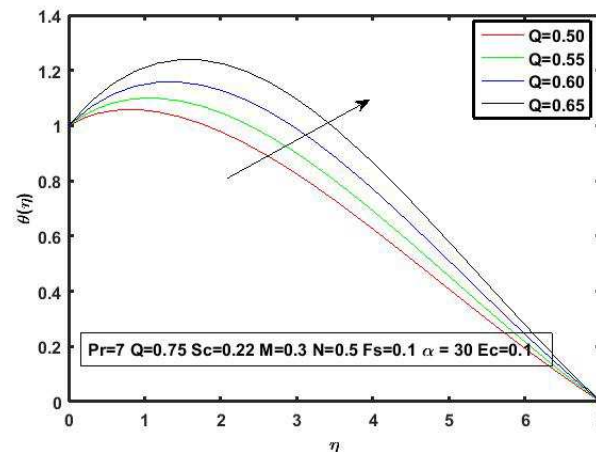


Figure 7: Temperature Profiles for Different Values of Heat Source

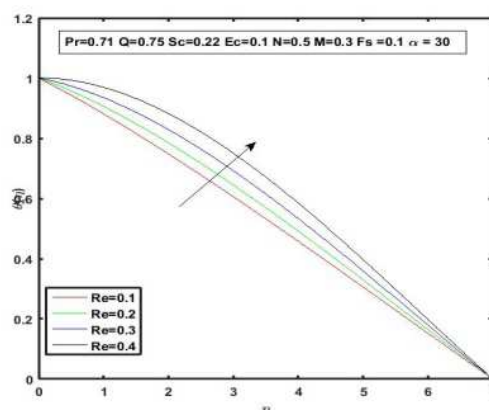


Figure 8: Temperature Profiles for Different Values of Reynolds Number

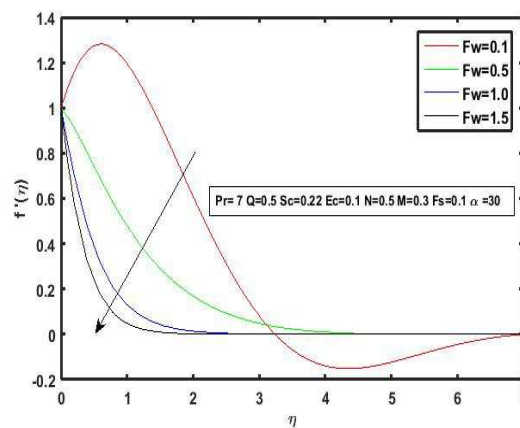


Figure 9: Temperature Profiles for Different Values of Suction Parameter

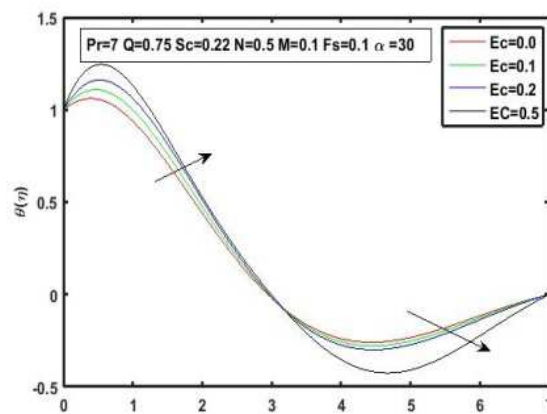


Figure 10: Temperature Profiles for Different Ekert Number

Figures 11-14 shows decrease of concentration profiles under the influence of chemical reaction parameter, Schmidt number, Reynolds number and suction parameter.

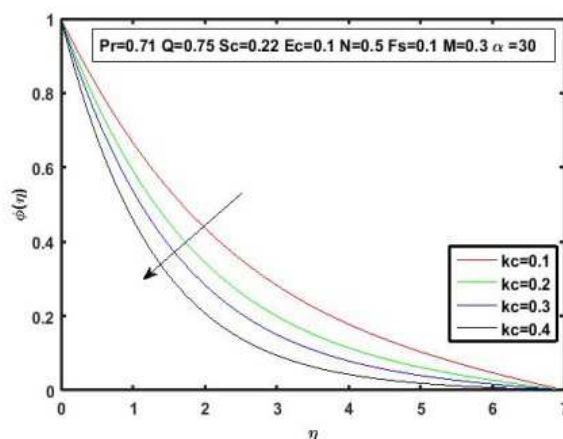


Figure 11: Concentration Profiles for Different Values of Chemical Reaction Parameter

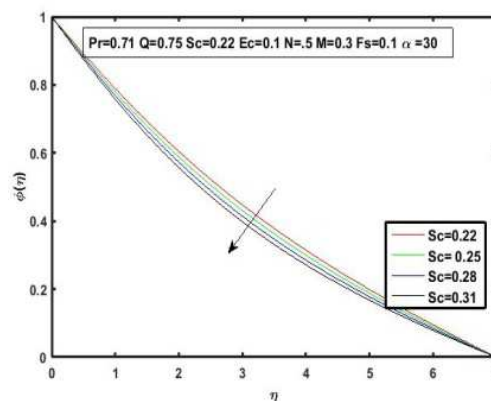


Figure 12: Concentration Profiles for Different Values of Schmidt Number

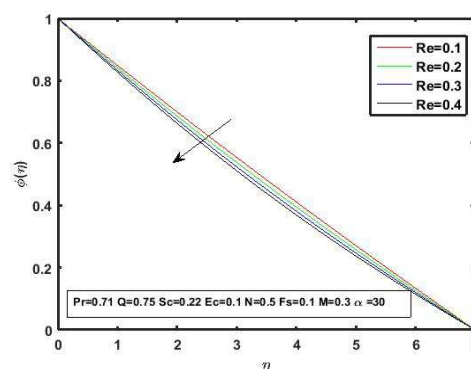


Figure 13: Concentration Profiles for Different Values of Reynolds Number

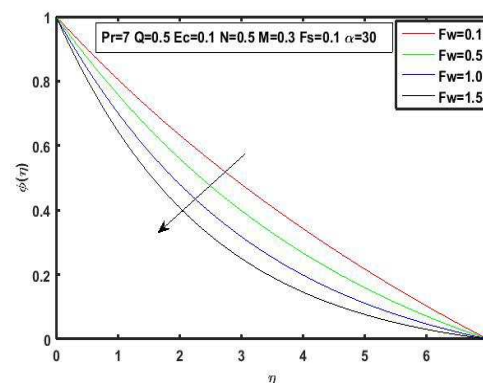


Figure 14: Concentration Profiles for Different Values of Suction Parameter

CONCLUSIONS

The present study is to analyze the influence of different physical parameters on the MHD free convective flow past inclined sheet. Similarity transformations used by Hasan et al[7] for velocity components along x – direction and y-direction are $u = Dxf'(\eta)$, $v = -\sqrt{D\nu} f(\eta)$. But in the present discussion, the transformations given by equation (9) contains Reynolds number. That is why, we have shown contribution of Reynolds number on velocity, temperature and concentration profiles. However, the results obtained by us are in agreement with those of Hasn et al[7]. The main purpose of attempting this problem is due to its vast application in industries, to determine the quality of the final product. ,

Finally, we can make the following conclusions of the present discussion.

- Velocity decreases with increase of Reynolds number and suction parameter. But as the angle of inclination porosity and magnetic parameter increases velocity increases up to $\eta = 2$ and decreases thereafter.
- Temperature decreases with increase of Radiation, suction parameter, but increases with increase of Heat source, Reynolds number and Ekert number.
- Concentration decreases with increase of Chemical reaction parameter, Schmidt number, Reynolds number and suction parameter.

Finally here, the comparative results of skin – friction ($f''(0)$), Nusselt number ($-\theta'(0)$) and Sherwood number ($-\phi'(0)$) are given under the influence of various parameters.

Present Results						
Pr	Q	Ec	N	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	0.50	0.1	0.5	3.3190	0.2721	0.4668
0.71	0.75	0.1	0.5	3.8393	0.2072	0.4801
7.0	0.75	0	0.5	2.3040	0.6987	0.3399
Results of Hasan et al						
0.71	0.50	0.1	0.5	3.40388	0.26724	0.46977
0.71	0.75	0.1	0.5	3.5041	0.19956	0.47801
7.0	0.75	0	0.5	2.25432	0.65081	0.33366

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